SOLUTION TO JEE Main - 2016

(Held on Sunday 3rd April, 2016)

PARADISE INSTITUTE

[A PREMIER INSTITUTE FOR JEE Main, JEE Advanced, NEET & Bihar CECE]

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PART – A (CHEMISTRY)

1. A stream of electrons from a heated filament was passed between two charged plates kept at a potential difference V esu. If e and m are charge and mass of an electron, respectively, then the value of h/λ (where λ is wavelength associated with electron wave) is given by:

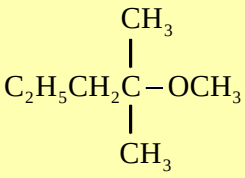
(1) 2meV (2) $\sqrt{meV}$ (3) $\sqrt{2meV}$ (4) meV

Ans. **(3)**

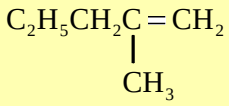
$$\lambda = \frac{h}{P}$$

$$\sqrt{2mev} = P = \frac{h}{\lambda}$$

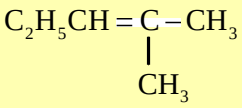
2. 2 – chloro -2 – methylpentane on reaction with sodium methoxide in methanol yields:



(a)



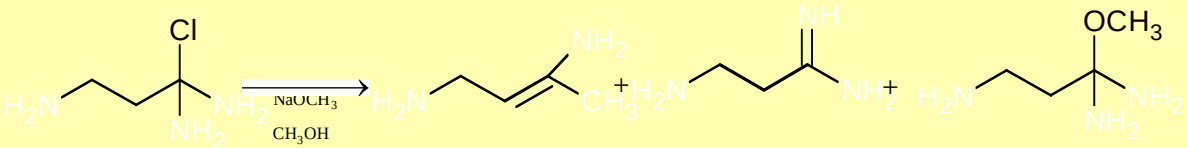
(b)



(c)

(1) (a) and (c) (2) (c) only (3) (a) and (b) (4) All of these

Ans. **(4)**



3. Which of the following compounds is metallic and ferromagnetic?

(1) CrO_2 (2) VO_2 (3) MnO_2 (4) TlO_2

Ans. **(1)**

CrO_2 is ferromagnetic in nature.

4. Which of the following statements about low density polythene is **FALSE**?

(1) It is a poor conductor of electricity.

(2) Its synthesis requires dioxygen or a peroxide initiator as a catalyst.

(3) It is used in the manufacture of buckets, dust – bins etc.

(4) Its synthesis requires high pressure.

Ans. **(3)**

Low density polyethylene is used for manufacture of squeeze; bottles; toys and flexible pipes.

5. For a linear plot of $\log(x/m)$ versus $\log p$ in a Freundlich adsorption isotherm, which of the following statements is correct? (k and n are constants)

(1) $1/n$ appears as the intercept.

(2) Only $1/n$ appears as the slope

(3) $\log(1/n)$ appears as the intercept

(4) Both k and $1/n$ appear in the slope term.

Ans. **(2)**

$$\frac{x}{m} = KP^{\frac{1}{n}}$$

$$\log\frac{x}{m} = \ln K + \frac{1}{n}\log P$$

$$n = C + mx$$

$$m = \frac{1}{n} = \text{slope}$$

6. The heats of combustion of carbon and carbon monoxide are -393.5 and -283.5 kJ mol^{-1}, respectively. The heat of formation (in kJ) of carbon monoxide per mole is:

(1) 676.5 (2) -676.5 (3) -110.5 (4) 110.5

Ans. **(3)**

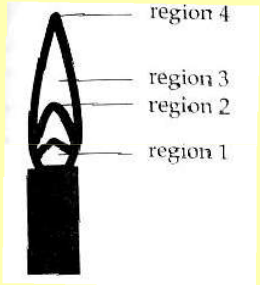
$C + O_2 \rightarrow CO_2$ $\Delta H_1 = -393.5$

$CO + \frac{1}{2}O_2 \rightarrow CO_2$ $\Delta H_2 = -283.5$

$C + \frac{1}{2}O_2 \rightarrow CO$ $\Delta H_f = \Delta H_1 - \Delta H_2$

$\Delta H_f = -110.0$

7. The hottest region of Bunsen flame shown in the figure below is:



(1) region 2 (2) region 3 (3) region 4 (4) region 1

Ans. **(1)**

8. Which of the following is an anionic detergent?

(1) Sodium lauryl sulphate (2) Cetyltrimethyl ammonium bromide

(3) Glyceryl oleate (4) Sodium stearate

Ans. **(1)**

Sodium lauryl sulphate

9. 18 g glucose $(C_6H_{12}O_6)$ is added to 178.2 g water. The vapor pressure of water (in torr) for this aqueous solution is:

(1) 76.0 (2) 752.4 (3) 759.0 (4) 7.6

Ans. **(2)**

$$\text{Moles of glucose} = \frac{18}{180} = \frac{1}{18} = \frac{1}{10} = 0.1$$

$$\text{Moles of water} = \frac{178.2}{18} = 9.9$$

$$X_{solute} = \frac{0.1}{0.1+9.9} = 0.01$$

$$\Delta = X_{solute}.P$$

$$= 0.01 \times 760 = 7.6 \text{ torr}$$

$$P_s = 760 - 7.6 = 752.4 \text{ torr}$$

10. The distillation technique most suited for separating glycerol from spent – lye in the soap industry is:

(1) Fractional distillation
(2) Steam distillation
(3) Distillation under reduced pressure
(4) Simple distillation

Ans. **(3)**

Glycerol can be separated from spent1ye in the soap industry by distillation under reduced pressure.

11. The species in which the N atom is in a state of sp hybridization is:

(1) NO_2^- (2) NO_3^- (3) NO_2 (4) NO_2^+

Ans. **(4)**

$[O = N = O]^+$ sp-hybridization

12. Decomposition of H_2O_2 follows a first order reaction. In fifty minutes the concentration of H_2O_2 decreases from 0.5 to 0.125 M in one such decomposition. When the concentration of H_2O_2 reaches 0.05 M, the rate of formation of O_2 will be:

(1) 6.93×10^{-4} mol min^{-1} (2) 2.66 L min^{-1} at STP
(3) 1.34×10^{-2} mol min^{-1} (4) 6.93×10^{-2} mol min^{-1}

Ans. **(1)**

$$T_{50} = 25 \text{ min}$$

$$R = \frac{\ln 2}{25} \text{ min}$$

$$r = R \times 0.05$$

$$\text{So rate of formation of } O_2 = \frac{1}{2} \times \frac{\ln 2}{25} \times 0.05$$

$$= 6.93 \times 10^{-4} \text{ mol min}^{-1}$$

13. The pair having the same magnetic moment is:
$[\text{At.No.}: Cr = 24, Mn = 25, Fe = 26, Co = 27]$

(1) $\left[Cr(H_2O)_6\right]^{2+}$ and $\left[Fe(H_2O)_6\right]^{2+}$ (2) $\left[Mn(H_2O)_6\right]^{2+}$ and $\left[Cr(H_2O)_6\right]^{2+}$

(3) $\left[CoCl_4\right]^{2-}$ and $\left[Fe(H_2O)_6\right]^{2+}$ (4) $\left[Cr(H_2O)_6\right]^{2+}$ and $\left[CoCl_4\right]^{2-}$

Ans. **(1)**

$Cr^{2+} = 4S^o 3d^4$ $Fe^{2+} = 4S^o 3d^6$

Unpaired $e^- = 4$ unpaired $e^- = 4$

$Mn^{2+} = 4S^o 3d^5$ $Cr^{2+} = 4S^o 3d^4$

Unpaired $e^- = 5$ unpaired $e^- = 4$

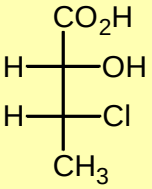
$Co^{2+} = 4S^o 3d^7$ $Fe^{2+} = 4S^o 3d^6$

Unpaired $e^- = 3$ unpaired $e^- = 4$

$Cr^{2+} = 4S^o 3d^4$ $Co^{2+} = 4S^o 3d^7$

unpaired $e^- = 4$ unpaired $e^- = 3$

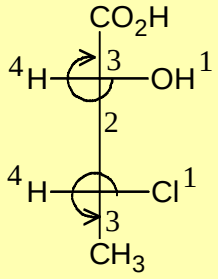
14. The absolute configuration of



Is

(1) $(2S, 3R)$ (2) $(2S, 3S)$ (3) $(2R, 3R)$ (4) $(2R, 3S)$

Ans. **(1)**



2S; 3R

15. The equilibrium constant at 298 K for a reaction $A + B \rightleftharpoons C + D$ is 100. If the initial concentration of all the four species were 1 M each, then equilibrium concentration of D (in mol L^{-1}) will be:

(1) 0.818 (2) 1.818 (3) 1.182 (4) 0.182

Ans. **(2)**

$A + B \rightleftharpoons C + D \qquad k_q = 100$

T = 0 1 1 1 1

at eq. 1-x 1-x 1+x 1+x

$$k_{eq} = 100 = \frac{(1+x)(1+x)}{(1-x)(1-x)} = \frac{[C][D]}{[A][B]}$$

$$\frac{1+x}{1-x} = 10$$

$$1+x = 10-10x$$

$$x = \frac{9}{11}$$

$$[D]_{eq} = 1+x = 1+\frac{9}{11} = \frac{20}{11} = 1.818\,\frac{mol}{lit}$$

16. Which one of the following ores is best concentration by froth floatation method?

(1) Siderite (2) Galena (3) Malachite (4) Magnetite

Ans. **(2)**

Galena (PbS) is sulphide ore therefore froth floatation is used.

17. At 300 K and 1 atm, 15 mL of a gaseous hydrocarbon requires 375 mL air containing 20% O_2 by volume for complete combustion. After combustion the gases occupy 330 mL. Assuming that the water formed is in liquid form and the volumes were measured at the same temperature and pressure, the formula of the hydrocarbon is:

(1) C_3H_8 (2) C_4H_8 (3) C_4H_8 (4) C_3H_6

Ans. **(Bonus)**

18. The pair in which phosphorous atoms have a formal oxidation state of +3 is

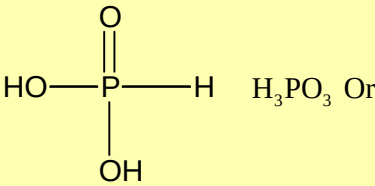
(1) Pyrophosphorous and hypophosphoric acids

(2) Orthophosphoric acids

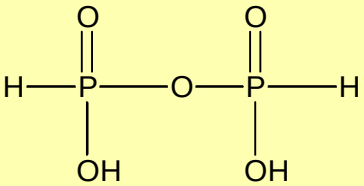
(3) Pyrophosphorous and pyrophosphoric acids

(4) Orthophosphorous and pyrophosphorous acids

Ans. **(4)**



H_3PO_3 Orthophosphorous Acid



$H_4P_2O_5$ pyrophosphorous Acid

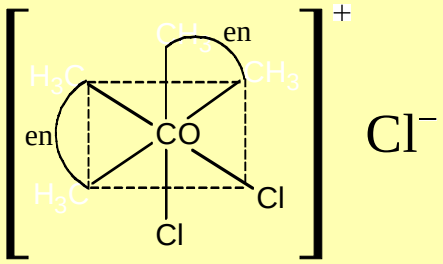
19. Which one of the following complexes shows optical isomerism?\

(1) cis $\left[Co(en)_2Cl_2\right]Cl$ (2) trans $\left[Co(en)_2Cl_2\right]Cl$

(3) $\left[Co(NH_3)_4Cl_2\right]Cl$ (4) $\left[Co(NH_3)_3Cl_3\right]$

(en = ethylenediamine)

Ans. **(1)**



No symmetry element

20. The reaction of zinc with dilute and concentrated nitric acid, respectively, produces:

(1) NO_2 and NO (2) NO and N_2O (3) NO_2 and N_2O (4) N_2O and NO_2

Ans. **(4)**

$4Zn + 10HNO_3(\text{dilute}) \rightarrow 4Zn(NO_3)_2 + 5H_2O + N_2O$

$Zn + 4HNO_3(\text{conc.}) \rightarrow Zn(NO_3)_2 + 2H_2O + 2NO_2$

21. Which one of the following statements about water is **FALSE**?

(1) Water can act both as an acid and as a base

(2) There is extensive intramolecular hydrogen bonding in the condensed phase.

(3) Ice formed by heavy water sinks in normal water.

(4) Water is oxidized to oxygen during photosynthesis.

Ans. **(2)**

Intermolecular H-bonding is present.

22. The concentration of fluoride, lead, nitrate and iron in a water sample from an underground lake was found to be 1000 ppb, 40 ppb, 100 ppm and 0.2 ppm, respectively. This water is unsuitable for drinking due to high concentration of:

(1) Lead (2) Nitrate (3) Iron (4) Fluoride

Ans. **(2)**

Nitrate

23. The main oxides formed on combustion of Li, Na and K in excess of air are, respectively:

(1) LiO_2, Na_2O_2 and K_2O (2) Li_2O_2, Na_2O_2 and KO_2

(3) Li_2O, Na_2O_2 and KO_2 (4) Li_2O, Na_2O and KO_2

Ans. **(3)**

$2Li + \frac{1}{2}O_2 \rightarrow Li_2O$ $2Na + O_2 \rightarrow Na_2O_2$

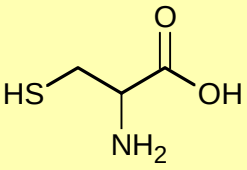
$K + O_2 \rightarrow KO_2$

Reactivity of alkali metals w.r.t. oxygen increases down to group.

24. Thiol group is present in:

(1) Cystine (2) Cysteine (3) Methionine (4) Cytosine

Ans. **(2)**



Cysteine

25. Galvanization is applying a coating of:

(1) Cr (2) Cu (3) Zn (4) Pb

Ans. **(3)**

Zinc coating applied on the surface of Iron. Zn being more electropositive gets oxidized

26. Which of the following atoms has the highest first ionization energy?

(1) Na (2) K (3) Sc (4) Rb

Ans. **(3)**

Scandium

27. In the Hoffmann bromide degradation reaction, the number of moles of NaOH and Br_2 used per mole of amine produced are :

(1) Four moles of NaOH and two moles of Br_2

(2) Two moles of NaOH and two moles of Br_2

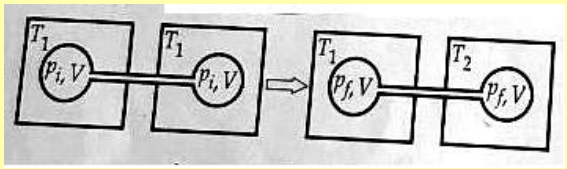
(3) Four moles of NaOH and one molesf of Br_2

(4) One moles of NaOH and one mole of Br_2

Ans. **(3)**

$$R-\overset{\overset{O}{\|}}{C}-NH_2+4NaOH+Br_2 \rightarrow R-NH_2+Na_2CO_3+2NaBr+2H_2O$$

28. Two closed bulbs of equal volume (V) containing an ideal gas initially at pressure p_i and temperature T_1 are connected through a narrow tube of negligible volume as sown in the figure below. The temperature of one of the bulbs is then raised to T_2. The final pressure p_f is :



(1) $2p_i\left(\frac{T_1}{T_1+T_2}\right)$ (2) $2p_i\left(\frac{T_2}{T_1+T_2}\right)$ (3) $2p_i\left(\frac{T_1T_2}{T_1+T_2}\right)$ (4) $p_i\left(\frac{T_1T_2}{T_1+T_2}\right)$

Ans. **(2)**

By conservation of moles

$$\frac{P_fV}{RT_1}+\frac{P_fV}{RT_2}=\frac{P_iV}{RT_1}+\frac{P_iV}{RT_1}$$

$$P_f = \frac{\frac{2P_i}{T_1}}{\frac{1}{T_1}+\frac{1}{T_2}}$$

$$P_f = \frac{2P_iT_2}{T_1+T_2}$$

29. The reaction of propene with HOCl $(Cl_2 + H_2O)$ proceeds through the intermediate:

(1) $CH_3 - CH^+ - CH_2 - Cl$ (2) $CH_3 - CH(OH) - CH_2^+$

(3) $CH_3 - CHCl - CH_2^+$ (4) $CH_3 - CH^+ - CH_2 - OH$

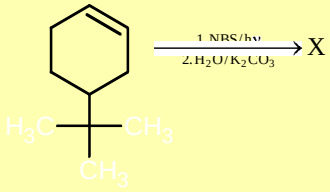
Ans. **(1)**

$$Cl_2 + H_2O \rightleftharpoons HOCl + HCl$$

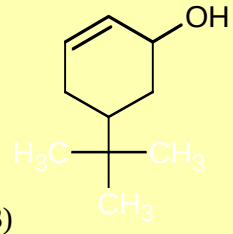
$$HO - Cl \rightarrow HO: + Cl^{\oplus}$$

$$CH_3 - CH = CH_2 + Cl^{\oplus} \rightarrow CH_3 - CH^{\oplus} - CH_2 - Cl$$

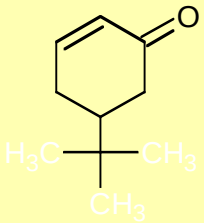
30. The product of the reaction given below is :



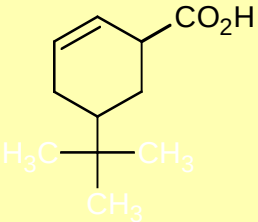
(1)



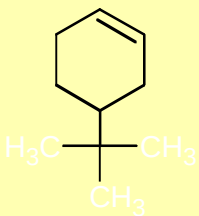
(2)



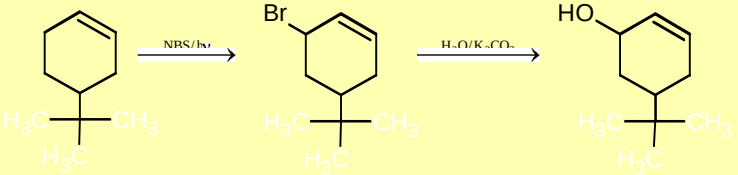
(3)



(4)



Ans. **(1)**

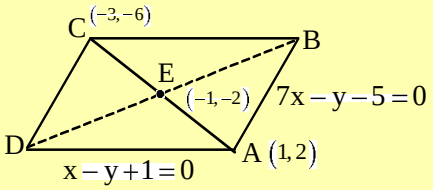


PART – B (MATHEMATCS)

31. Two sides of a rhombus are along the lines, $x-y+1=0$ and $7x-y-5=0$. If its diagonals intersect at $(-1,-2)$, then which one of the following is a vertex of this rhombus?

(1) $(-3,-8)$ (2) $\left(\frac{1}{3},-\frac{8}{3}\right)$ (3) $\left(-\frac{10}{3},-\frac{7}{3}\right)$ (4) $(-3,-9)$

Sol: **(2)**



$7x-y-5=0$

$x-y+1=0$

$6x-6=0$

$x=1$

$7-y-5=0$

$y=2$

E is the mid-point of AC

$C\equiv(2(-1)-1,-2(2)-2)\equiv(-3,-6)$

Equation of $BC\equiv y+6=x+3$

$y=x-3$

Equation of $CD\equiv y+6=7(x+3)$

$y=7x+15$

Solution BC & AB we get

$x=\frac{1}{3},y=\frac{-8}{3}$

32. If the 2nd, 5th and 9th terms of a non-constant A.P. are in G.P., t hen the common ratio of this G.P. is:

(1) $\frac{4}{3}$ (2) 1 (3) $\frac{7}{4}$ (4) $\frac{8}{5}$

Sol: **(1)**

Let A.P be a, a + d, a + 2d, ……

$\therefore$ a + d, a + 4d, a + 8d are in G.P

$\Rightarrow (a+4d)^2=(a+d)(a+8d)$

$\Rightarrow a^2+16d^2+8ad=a^2+9ad+8d^2$

$\Rightarrow 8d^2=ad$

$\Rightarrow \quad =8d$

$\therefore$ common Ratio $=\frac{a+4d}{a+d}=\frac{12d}{9d}$

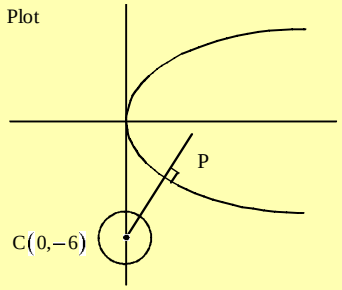
$=\frac{4}{3}$

33. Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle, $x^2 + (y+6)^2 = 1$. Then the equation of the circle, passing through C and having its centre at P is:

(1) $x^2 + y^2 - x + 4y - 12 = 0$ (2) $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$

(3) $x^2 + y^2 - 4x + 9y + 18 = 0$ (4) $x^2 + y^2 - 4x + 8y + 12 = 0$

Sol: **(4)**



Let P be $(2t^2, 4t)$

$\therefore$ Normal at P passes through centre of circle $C(0,-6)$

Normal is

$y + xt = 4t + 2t^3$

Put $(0,-6)$

$-6 = 4t^2 + 2t^3$

$\Rightarrow t^3 + 2t^2 + 3 = 0$

$t = -1$

$\therefore$ circle with centre P & passing through C is

$x^2 + y^2 - 4x + 8y + 12 = 0$

34. The system of linear equations

$x + \lambda y - z = 0$

$\lambda x - y - z = 0$

$x + y - \lambda z = 0$

Has a non-trivial solution for:

(1) exactly one value of λ (2) exactly two values of λ.

(3) exactly three values of λ (4) infinitely many values of λ.

Sol: **(3)**

For non-trivial solution

$$\begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$\Rightarrow (\lambda+1) - \lambda(-\lambda^2+1) - 1(\lambda+1) = 0$

$\Rightarrow \lambda^3 = \lambda$

$\Rightarrow \lambda = 0, \pm 1$

35. If $f(x)+2f\left(\frac{1}{x}\right)=3x,\ x\neq 0$, and $s=\{x\in R: f(x)=f(-x)\}$; then S:

(1) contains exactly one element.
(2) contains exactly two elements.
(3) contains more than two elements.
(4) in an empty set.

Sol: **(2)**

$f(x)+2f\left(1/x\right)=3x$

Replacing x by $\frac{1}{x}$

$f\left(1/x\right)+2f(x)=\frac{3}{x}$

$\Rightarrow$ Solving $-3f(x)=3x-\frac{6}{x}$

$f(x)=-\ +\frac{2}{x}$

Now $f(x)=f(-x)$

$-x+\frac{2}{x}=x-\frac{2}{x}$

$2x=\frac{4}{x}\Rightarrow x^2=2$

$x=\pm\sqrt{2}$

Hence, two value of x.

36. Let $p=\lim\limits_{x\to 0^+}\left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$ then log p is equal to:

(1) 1 (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) 2

Sol: **(2)**

$P=\lim\limits_{x\to 0^+}\left(1+\tan^2\sqrt{x}\right)\frac{1}{2x}$

It is from 1^∞

$\therefore P=e^{\lim\limits_{x\to 0^+}\left(\tan^2\sqrt{x}\right)\times\frac{1}{2x}}$

$=e^{\lim\limits_{x\to 0^+}\left(\frac{\tan^2\sqrt{x}}{\left(\sqrt{x}\right)^2}\times\frac{1}{2}\right)}$

$P=\ e^{\frac{1}{2}}$

$\log P=\frac{1}{2}$

37. A value of θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is:

(1) $\frac{\pi}{6}$ (2) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (3) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (4) $\frac{\pi}{3}$

Sol: **(3)**

Rationalize complex number

$$\frac{(1+3i\sin\theta)(1-2i\sin\theta)}{1+4\sin^2 Q}$$

$\therefore$ put real part = 0

$$2-6\sin^2\theta=0$$

$$\sin\theta=\pm\frac{1}{\sqrt{3}}$$

38. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is:

(1) $\frac{4}{\sqrt{3}}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\sqrt{3}$ (4) $\frac{4}{3}$

Sol: **(2)**

$$\frac{2b^2}{a}=8$$

$$2b=\frac{1}{2}(2ae)$$

$$2b=ae$$

$$\Rightarrow a^2e^2=a^2+b^2$$

$$a^2e^2=a^2+\frac{a^2e^2}{4}$$

$$e^2=1+\frac{e^2}{4}$$

$$3\frac{e^2}{4}=1$$

$$e=\frac{2}{\sqrt{3}}$$

39. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true?

(1) $3a^2-32a+84=0$ (2) $3a^2-34a+91=0$

(3) $3a^2-23a+44=0$ (4) $3a^2-26a+55=0$

Sol: **(1)**

$$\sigma^2=\frac{7}{2}=\sqrt{\left(E(x)^2\right)-\left(E(x)\right)^2}$$

$$\frac{49}{4}=\frac{2^2+3^2+a^2+11^2}{4}-\left(\frac{16+a}{4}\right)^2$$

40. The integral $\int \frac{2x^{12}+5x^9}{\left(x^5+x^3+1\right)^3}\,dx$ is equal to:

(1) $\frac{x^{10}}{2\left(x^5+x^3+1\right)^2}+C$ (2) $\frac{x^5}{2\left(x^5+x^3+1\right)^2}+C$

(3) $\frac{-x^{10}}{2\left(x^5+x^3+1\right)^2}+C$ (4) $\frac{-x^5}{\left(x^5+x^3+1\right)^2}+C$

Sol: **(1)**

$$\int \frac{2x^{12}+5x^9}{\left(x^5+x^3+1\right)^3}dx$$

$$\int \frac{\frac{2}{x^3}+\frac{5}{x^6}}{\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^3}dx$$

Put $1+\frac{1}{x^2}+\frac{1}{x^5}=t$

$$\left(\frac{2}{x^3}\ \frac{5}{x^6}\right)dx=dt$$

$$=\int \frac{-dt}{t^3}=\frac{1}{2t^2}+C$$

$$\Rightarrow \frac{x^{10}}{2\left(x^5+x^3+1\right)^2}+C$$

41. If the line, $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z+4}{3}$ lies in the plane, $lx+my-z=9$, then l^2+m^2 is equal to:

(1) 18 (2) 5 (3) 2 (4) 26

Sol: **(3)**

$\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z+4}{3}$ (given line)

$lx+my-z=9$ (given plane)

$\because$ (3, -2, -4) lies in plane

So, $3\ell-2m+4=9$ …(i)

& $2\hat{i}-\hat{j}+3\hat{k}$ and $\ell\hat{i}+m\hat{j}-\hat{k}$

Are perpendicular vectors.

$2\ell-m-3=0$ …(ii)

From (i) & (ii)

$\ell=1;\ m=-1$

$\Rightarrow \ell^2+m^2=2$

42. If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$, is:

(1) 5 (2) 7 (3) 9 (4) 3

Sol: **(2)**

$\cos x + \cos 2x + \cos 3x + \cos 4x = 0$

$0 \le x < 2\pi$

$(\cos x + \cos 4x) + (\cos 2x + \cos 3x) = 0$

$2\cos\frac{5x}{2}\cos\frac{3x}{2} + 2\cos\frac{5x}{2}\cos\frac{x}{2} = 0$

$2\cos\frac{5x}{2}\left(\cos\frac{3x}{2} \quad \cos\frac{x}{2}\right) = 0$

$2\cos\frac{5x}{2} \times 2\cos x\cos\frac{x}{2} = 0$

$\cos\frac{x}{2} = 0$ $\cos x = 0$ $\cos\frac{5x}{2} = 0$

$x = \pi$ $x = \frac{\pi}{2}, \frac{3\pi}{2}$ $x = \frac{\pi}{5}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{9\pi}{5}$

$\rightarrow$ 7 Distinct solutions

43. The area (in sp. units) of the refion $\{(x, y): y^2 \ge 2x \text{ and } x^2 + y^2 \le 4x, x \ge 0, y \ge 0\}$ is:

(1) $\pi - \frac{8}{3}$ (2) $\pi - \frac{4\sqrt{2}}{3}$ (3) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$ (4) $\pi - \frac{4}{3}$

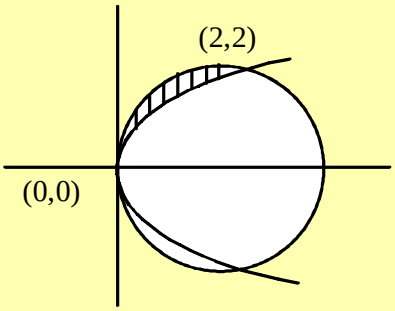
Sol: **(1)**

$C_1 : (x-2)^2 + y^2 \le 4$

$C_2 : y^2 \ge 2x$

Solve C_1 & C_2

To get integer $x = 0, x = 2$



$A = \left[\int_0^2 \left(\sqrt{4 - (x-2)^2} - \sqrt{2x}\right) dx\right]$

$$= \left[\int_0^2 \frac{(x-2)}{2}\sqrt{4-(x-2)^2} + \sin^{-1}\frac{x-2}{2} - \frac{\sqrt{2}x^{3/2}}{\frac{3}{2}} \right]$$

$$= \left[\frac{-2\sqrt{2}}{3} \times 2^{3/2} + \pi \right]$$

$$= \pi - \frac{8}{3}$$

44. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is:

(1) $\frac{\pi}{2}$ (2) $\frac{2\pi}{3}$ (3) $\frac{5\pi}{6}$ (4) $\frac{3\pi}{4}$

Sol: **(3)**

$$\bar{a} \times (\bar{b} \times \bar{c}) = \frac{\sqrt{3}}{2}(\bar{b} + \bar{c})$$

$$(\bar{a} \cdot \bar{c})\bar{b} - (\bar{a} \cdot \bar{b})\bar{c} = \frac{\sqrt{3}}{2}\bar{b} + \frac{\sqrt{3}}{2}\bar{c}$$

$$\Rightarrow \bar{a} \cdot \bar{c} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \bar{a} \cdot \bar{b} = \frac{-\sqrt{3}}{2} \Rightarrow |\bar{a}||\bar{b}|\cos\theta = \frac{-\sqrt{3}}{2}$$

$$1 \times 1 \times \cos\theta = \frac{-\sqrt{3}}{2}$$

$$\Rightarrow \theta = \frac{5\pi}{6}$$

45. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side = x units and a circle of radius = r units. If the sum of the areas of the square and the circle so formed is minimum, then:

(1) $(4-\pi)x = \pi r$ (2) $x = 2r$ (3) $2x = r$ (4) $2x = (\pi + 4)r$

Sol: **(2)**

P, $2 - P$

$$x = \left(\frac{p}{4}\right) \qquad r = \left(\frac{2-p}{2\pi}\right)$$

$$A = \frac{p^2}{16} + \frac{\pi(2-p)^2}{4\pi^2}$$

$$\frac{dA}{dP} = \frac{2p}{168} + \frac{2(2-p)(-1)}{2\pi}$$

$$= \frac{p}{8} + \frac{p}{2\pi} - \frac{1}{\pi} = 0$$

$$\frac{p}{8} = \frac{\pi}{\pi + 4}$$

$$\Rightarrow \frac{x}{2} = r$$

$$x = 2r$$

46. The distance of the point $(1, -5, 9)$ from the plane x – y + z = 5 measured along the line x = y = z is:

(1) $10\sqrt{3}$ (2) $\frac{10}{\sqrt{3}}$ (3) $\frac{20}{3}$ (4) $3\sqrt{10}$

Sol: **(1)**

Let $P(1+\lambda, -5+\lambda, 9+\lambda)$

It lies on plane

$$x - y + z = 5$$

$$\Rightarrow \quad 1+\lambda+5-\lambda+9+\lambda = 5$$

$$\lambda = -10$$

Distance $= \sqrt{3}|\lambda|$

$$= 10\sqrt{3}$$

47. If a curve y = f(x) passes through the point (1,-1) and satisfies the differential equation, $y(1+xy)dx = x\ dy$, then $f\left(-\frac{1}{2}\right)$ is equal to:

(1) $-\frac{4}{5}$ (2) $\frac{2}{5}$ (3) $\frac{4}{5}$ (4) $-\frac{2}{5}$

Sol: **(3)**

$$y\,dx - x\,dy = -xy^2 dx$$

$$\frac{ydx - x\,dy}{y^2} = -x\ dx$$

Integrating

$$\frac{x}{y} = -\frac{x^2}{2} + C$$

It passes thorough (1, -1)

$$-1 = -\frac{1}{2} + C \Rightarrow C = -\frac{1}{2}$$

Then $f\left(\frac{-1}{2}\right) = \frac{4}{5}$

48. If the number of terms in the expansion of $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^n$, $x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion is:

(1) 2187 (2) 243 (3) 729 (4) 64

Sol: **(3)**

$^{n+3-1}C_{3-1} = 28$

$\Rightarrow n = 6$

i.e $\Rightarrow \left(1-\frac{2}{x}+\frac{4}{x^2}\right)^6 = a$

sum of coefficient $= (3)^6$ [put x = 1]

= 729

49. Consider $f(x) = \tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right), x \in \left(0, \frac{\pi}{2}\right)$

A normal to $y = f(x)$ at $x = \frac{\pi}{6}$ also passes through the point:

(1) $\left(0, \frac{2\pi}{3}\right)$ (2) $\left(\frac{\pi}{6}, 0\right)$ (3) $\left(\frac{\pi}{4}, 0\right)$ (4) (0, 0)

Sol: **(1)**

$$f(x) = \tan^{-1}\sqrt{\frac{1+\cos\left(\frac{\pi}{2}-x\right)}{1-\cos\left(\frac{\pi}{2}+x\right)}}$$

$$= \tan^{-1}\sqrt{\frac{1-\cos\left(\frac{\pi}{2}+x\right)}{1+\cos\left(\frac{\pi}{2}+x\right)}}$$

$$= \tan^{-1}\left|\tan\left(\frac{\pi}{4}\ \frac{x}{2}\right)\right|$$

$$y = \frac{\pi}{4}+\frac{x}{2}$$

$$\frac{dy}{dx} = \frac{1}{2}$$

Slope of normal = - 2

At $x = \frac{\pi}{6}$, $y = \frac{\pi}{3}$

Equation of normal is

$$\left(y-\frac{\pi}{3}\right) = -2\left(x-\frac{\pi}{6}\right)$$

At x = 0 $y = \frac{2\pi}{3}$

50. For $x \in R$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then:

(1) $g'(0) = \cos(\log 2)$

(2) $g'(0) = -\cos(\log 2)$

(3) g is differentiable at x = 0 and $g'(0) = \sin(\log 2)$

(4) g is not differentiable at x = 0

Sol: **(1)**

$f(x) = |\ln 2 - \sin x|$

Near about x = 0

$f(x) = \ln 2 - \sin x$

$g(x) = f(f(x)) = \ln 2 - \sin(\ln 2 - \sin x)$

$g'(x) = -\cos(\ln 2 - \sin x)(-\cos x)$

$g'(0) = -\cos(\ln 2 - 0)(-1)$

$g'(0) = \cos(\ln 2)$

51. Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is NOT true?

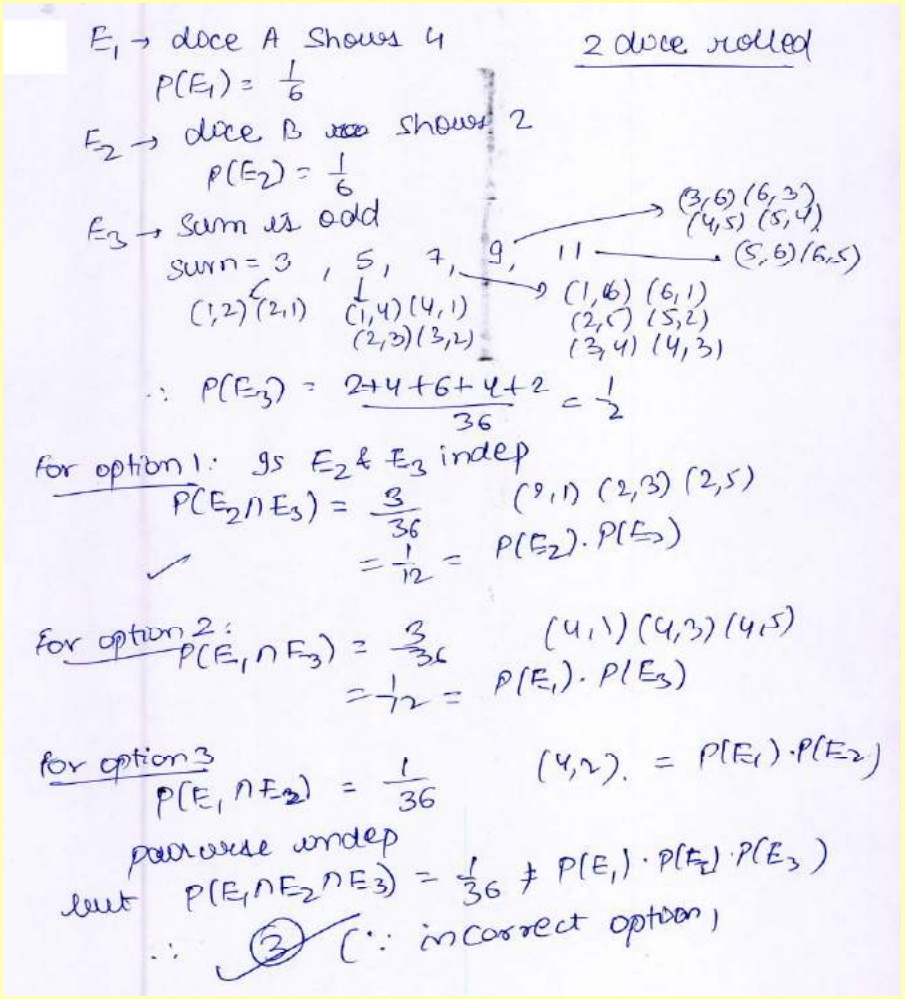
(1) E_2 and E_3 are independent.

(2) E_1 and E_3 are independent.

(3) E_1, E_2 and E_3 are independent.

(4) E_1 and E_2 are independent.

Sol: **(3)**



52. If $A=\begin{bmatrix}5a & -b\\ 3 & 2\end{bmatrix}$ and A adj A = A A^T, then 5a + b is equal to:

(1) 5 (2) 4 (3) 13 (4) – 1

Sol: **(1)**

$AA^T = A\,adj\,A \quad |A| = 10a+3b$

$= |A| I$

$\begin{pmatrix}5a & -b\\ 3 & 2\end{pmatrix}\begin{pmatrix}5a & 3\\ -b & 2\end{pmatrix}=\begin{pmatrix}10a+3b & 0\\ 0 & 10a+3b\end{pmatrix}$

$\begin{pmatrix}25a^2+b^2 & 15a-2b\\ 15a-2b & 9+4\end{pmatrix}=\begin{pmatrix}10a+3b & 0\\ 0 & 10a+3b\end{pmatrix}$

$15a = 2b \qquad 25a^2+b^2 = 10a+3b$

$b=\frac{15}{2}a \qquad 25a^2+\frac{225a^2}{4}=10a+\frac{45a}{2}$

$\frac{65a^2}{4}=\frac{13a}{2}$

$a = 2/5$

& $b = 3$

$5a+b=5$

Ans:-1

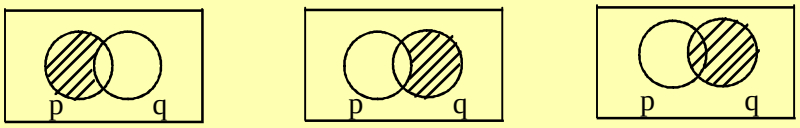
53. The Boolean Expression $(p\wedge\sim q)\vee q\vee(\sim p\wedge q)$ is equivalent to:

(1) $p\wedge q$ (2) $p\vee q$ (3) $p\vee\sim q$ (4) $\sim p\wedge q$

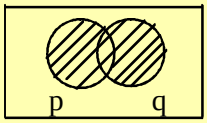
Sol: **(2)**

$(P\wedge\sim q)\vee q\vee(\sim p\wedge q)$

$p\wedge\sim q$ $\sim p\wedge q$ q



Taking union we get



54. The sum of all real values of x satisfying the equation $(x^2-5x+5)^{x^2+4x-60}=1$ is:

(1) -4 (2) 6 (3) 5 (4) 3

Sol: **(4)**

$(x^2-5x+5)^{x^2+4x-60}=1$

In a^x,

When $x=0$ $\therefore x^2+4x-60=0$

$\therefore (x+10)(x-6)=0$

$x=-10,6$

When $a=1$ $\rightarrow x^2-5x+5=1$

$x^2-5x+4=0$

$(x-1)(x-4)=0$

$x=1,4$

When $a=-1$ $\rightarrow x^2-5x+5=-1$

$x^2-5x+5=-1$

$x^2-5x+6=0$

$x=2,3$

For x = 2 , the exponent $x^2+4x-60$ becomes

$4+8-60=-48$

And for x = 3, the exponent becomes $9+12-60=-39$

So Acceptable values are $-10,6,1,4,2$

So sum of roots = 3

55. The centres of those circles which touch the circle, $x^2+y^2-8x-8y-4=0$, externally and also touch the x-axis, lie on:

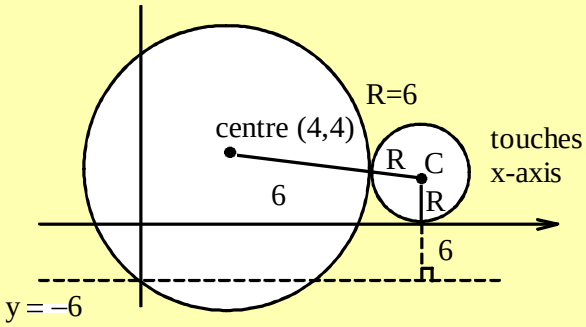
(1) an ellipse which is not a circle.

(2) a parabola.

(3) a parabola.

(4) a circle.

Sol: **(3)**



Centre is equidistant from (4,4) & the line $y=-6$

Locus is parabola

56. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is:

(1) 59^{th} (2) 52^{th} (3) 58^{th} (4) 46^{th}

Sol: **(3)**

Rank of Small

Alphabetical order $\rightarrow A, L, L, M, S$

Letter starting with $A \rightarrow \frac{4!}{2!}=12$

Letter starting with $L \rightarrow 4!=24$

Letter starting with $M \rightarrow \frac{4!}{2!}=12$

Letter starting with $SA \rightarrow \frac{3!}{2!}=3$

Letter starting with $SL \rightarrow 3!=-6$

Letter starting with SMALL $\rightarrow 58$

$\therefore$ 58th rank

57. $\lim\limits_{x\to\infty}\left(\frac{(n+1)(n+2)...3n}{n^{2n}}\right)^{1/n}$ is equal to:

(1) $\frac{27}{e^2}$ (2) $\frac{9}{e^2}$ (3) $3\log 3-2$ (4) $\frac{18}{e^4}$

Sol: **(1)**

$$\Rightarrow \ln L=\frac{1}{n}\left(\ln\left(1+\frac{1}{n}\right)+\ln\left(1+\frac{2}{n}\right)+.....+\ln\left(1+\frac{2n}{n}\right)\right)$$

$$= \frac{1}{n}\sum_{l=1}^{3n} \ln\left(1+\frac{l}{n}\right)$$

$$= \int_0^2 \ln(1+x)$$

$$= \int_1^3 \ln x = x\ln x - x\Big|_1^3$$

$$= 3\ln 3 - 2$$

$$\Rightarrow L = \frac{27}{e^2}$$

58. If the sum of the first ten terms of the series

$$\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + \ldots\ldots,$$

Is $\frac{16}{5}m$, then m is equal to:

(1) 101 (2) 100 (3) 99 (4) 102

Sol: **(1)**

$$= \frac{4^2}{5^2}$$

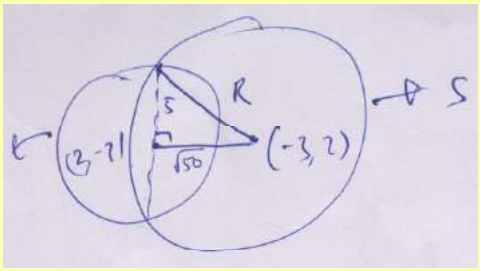
$$= \frac{4^2}{5^2}\left(\frac{11\times 12\times 23}{6} - 1\right) = \frac{16}{5}m$$

$$\Rightarrow \quad = 101$$

59. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is at (-3, 2), then the radius of S is:

(1) $5\sqrt{3}$ (2) 5 (3) 10 (4) $5\sqrt{2}$

Sol: **(1)**



Centre of s (-3, 2)

Here by Pythagoras theorm

$R^2 = 5^2 + 50$

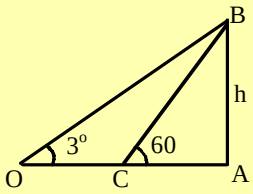
$R^2 = 75$

$R = 5\sqrt{3}$

60. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30^o. After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60^o. Then the time taken (in minutes) by him, from B to reach the pillar, is:

(1) 10 (2) 20 (3) 5 (4) 6

Sol: **(3)**



$\tan 60^o = \sqrt{3} = \frac{h}{CA}$

$OC = h\sqrt{3} - \frac{h}{\sqrt{3}} \rightarrow \frac{2h}{\sqrt{3}}$

$\frac{2h}{\sqrt{3}} \Rightarrow \upsilon \times 10$ …(1)

$\frac{h}{\sqrt{3}} = \upsilon \times t$ …(2)

$\Rightarrow 2 = \frac{10}{t}$

$t = 5$

PART – C (PHYSICS)

61. A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is (take g = 10ms^{-2})

(1) 2s (2) $2\sqrt{2}$s (3) $\sqrt{2}$s (4) $2\pi\sqrt{2}$s

Ans. (2)

$$\frac{dy}{dt}=\sqrt{\frac{g\,y\rho A}{\mu}}$$

$$\frac{dy}{dt}=\int\sqrt{gy}$$

$$\int\frac{dy}{\sqrt{y}}=\sqrt{g}\,dt$$

$$\left.\frac{y^{-1/2+1}}{-1/2+1}\right|_0^{\ell}=\sqrt{g}\,t\Big|_0^t$$

$$t=2\sqrt{\frac{20}{10}}$$

$$=2\sqrt{2}\ \text{sec}$$

62. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up ? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take g = 9.8 ms^{-2}

(1) 6.45×10^{-3} kg (2) 9.89×10^{-3} kg (3) 12.89×10^{-3} kg (4) 2.45×10^{-3} kg

Ans. (3)

Work done by the man = change in potential energy

$\therefore$ $w = mgh\times1000$

$= 98,000$ J

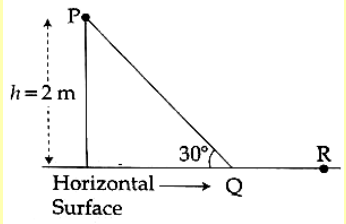
Now, assume x kg of fat is required

$\therefore$ $0.2\times x\times3.8\times10^7 = 98,000$

$\therefore$ $x = 12.89\times10^{-3}$kg

63. A point particle of *m*, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction between the particle and the rough track equals μ. The particle is released, form the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR of the track, are equals to each other, and no energy is lost when particle changes direction from PQ to QR.

The value of the coefficient of friction μ and the distance x (=QR), are, respectively close to;



(1) 0.2 and 3.5 m (2) 0.29 m and 3.5 m

(3) 0.29 and 6.5 m (4) 0.2 m and 6.5 m

Ans. (2)

$0 = mgh - \mu mg\cos\theta\frac{2}{\sin\theta} - \mu mgx$

$h = \mu \quad + \mu \quad \theta\frac{2}{\sin\theta}$

$h = \mu x + 2\mu\sqrt{3}$

$\mu = \frac{1}{2\sqrt{3}}$

$\mu = 0.289$

X = 3.5 m

64. Two identical wires A and B, each of length '*l*', carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side '*a*'. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is:

(A) $\frac{\pi^2}{16\sqrt{2}}$ (2) $\frac{\pi^2}{16}$ (3) $\frac{\pi^2}{8\sqrt{2}}$ (4) $\frac{\pi^2}{8}$

Ans. (3)

$B_A = \frac{\mu_0 i}{2R}$

$B_A = \frac{\mu \cdot \pi}{\ell} = \frac{\mu_0 i}{\ell}\pi$

$B_B = \left\{\frac{\mu_0 i}{4\pi\left(a/2\right)}.\left\{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right\}\right\} \times 4$

$B_B = \frac{\mu_0 i}{2\pi a}.\sqrt{2}.4$

$B_B = 2\sqrt{2}\frac{\mu_0 i}{\pi a}$

$B_B = \frac{\mu_0 i}{\pi\ell}.8\sqrt{2} = \frac{\mu_0 i}{\ell}.\frac{8\sqrt{2}}{\pi}$

65. A galvanometer having a coil resistance of $100\,\Omega$ gives a full scale deflection, when a current of 1 mA is passed through it. The value of the resistance, which can convert this galvanometer into a ammeter giving a full scale deflection for a current of 10 A, is

(1) $2\,\Omega$ (2) $0.1\,\Omega$ (3) $3\,\Omega$ (4) $0.01\,\Omega$

Ans. (4)

Here $V_S = V_G$

$i_g.G = \left(i - i_g\right).S$

Given $i_g = 1\text{mA} = 10^{-3}\text{A}$

G = Ω

i = 10 A

$10^{-3}\left(100\right) = \left(10 - 10^{-3}\right).S$

$S = 10^{-2}\,\Omega$

66. An observer looks at a distant tree of height 10 m with a telescope of magnifying power of 20. To the observer the tree appears:
(1) 10 times nearer (2) 20 times taller
(3) 20 times nearer (4) 10 time taller

Ans. (3)

$$\text{Magnification} = \frac{\text{final visual angle}}{\text{intial visual angle}}$$

$$20 = \frac{\left(\frac{\text{final height}}{\text{final distance}}\right)}{\left(\frac{\text{initial height}}{\text{inital distance}}\right)}$$

It is obvious that the tree is not going to look 200 m tall.

$\therefore$ Initial height = final height

$$\therefore 20 = \frac{\text{inital distance}}{\text{final distnace}}$$

$\therefore$ 20 times nearer

67. The temperature dependence of resistance of Cu and undoped Si in the temperature range 300–400 K, is best described by:
(1) Linear increases for Cu, exponential increases Si
(2) Linear increases for Cu, exponential decreases Si
(3) Linear decreases for Cu, decreases Si
(4) Linear increases for Cu, increases Si

Ans. (2)

68. Choose the correct statement:
(1) In amplitude modulation the frequency of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.
(2) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.
(3) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the frequency of the audio signal.
(4) In amplitude modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

Ans. (2)

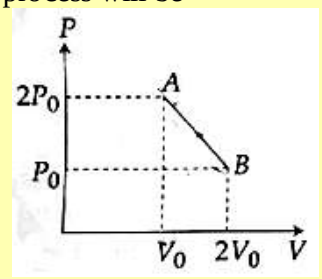
69. Half–lives of two radioactive elements A and B are 20 minutes and 40 minutes, respectively. Initially, the samples have equal number of nuclei. After 80 minutes, the ratio of decayed numbers of A and B nuclei will be:
(1) 4 :1 (2) 1 : 4 (3) 5 : 4 (4) 1 : 16

Ans. (3)

No. of nuclei decayed = $N_0\left(1-2^{-t/t_{1/2}}\right)$

$\therefore$ required ratio = $\frac{5}{4}$

70. '*n*' moles of an ideal gas undergoes a process $A \rightarrow Bs$ as shown in the figure. The maximum temperature of the gas during the process will be



(1) $\frac{3\ P_0V_0}{2nR}$ (2) $\frac{9\ P_0V_0}{2nR}$ (3) $\frac{9\ P_0V_0}{nR}$ (4) $\frac{9\ P_0V_0}{4nR}$

Ans. (4)

$$P = -\frac{P_0}{V_0}V + 3P_0$$

$$\frac{nRT}{V} = -\frac{P_0}{V_0}V + \frac{3P_0V}{nR}$$

$$T = -\frac{P_0}{nRV_0}V^2 + \frac{3P_0V}{nR} \quad \ldots (i)$$

For maximum temperature,

$$\frac{dT}{dV} = -\frac{2P_0}{nRV_0}v + \frac{3P_0}{nR} = 0$$

$$V = \frac{3V_0}{2} \quad \ldots (ii)$$

Putting value of equation (ii) in (i)

$$T_{max} = \frac{9P_0V_0}{4nR}$$

71. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work, is close to

(1) 0.08 H (2) 0.044 H (3) 0.065 H (4) 80 H

Ans. (3)

$v = iR \Rightarrow 80 = 10R$

$R = \ \Omega$

For AC circuit

$$\frac{220}{\sqrt{8^2 + (2\pi \times 50L)^2}} = 10A$$

$\Rightarrow$ L = 0.065 H

72. A pipe open at both ends has a fundamental frequency *f* in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now:

(1) $\frac{3f}{4}$ (2) $2f$ (3) f (4) $\frac{f}{2}$

Ans. (3)

$$f_0 = \frac{V}{2\ell}$$

$$f_0' = \frac{V}{4\left(\frac{L}{2}\right)}$$

$$f_0' = f_0$$

73. The box of a pin hole camera, of length L, has a hole of radius *a*. It is assumed that when the hole is illuminated by a parallel beam of light of wavelength λ the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size (say b_{min}) when

(1) $a = \sqrt{\lambda L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$ (2) $a = \sqrt{\lambda L}$ and $b_{min} = \sqrt{4\lambda L}$

(3) $a = \frac{\lambda^2}{L}$ and $b_{min} = \sqrt{4\lambda L}$ (4) $a = \frac{\lambda^2}{L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$

Ans. (2)

$$\text{extra spread} = \ell\frac{\lambda}{a}$$

$$\therefore \text{Total} = a + \frac{\ell\lambda}{a}$$

$$\frac{a + \frac{\ell\lambda}{a}}{2} \geq \sqrt{a \times \frac{\ell\lambda}{a}}$$

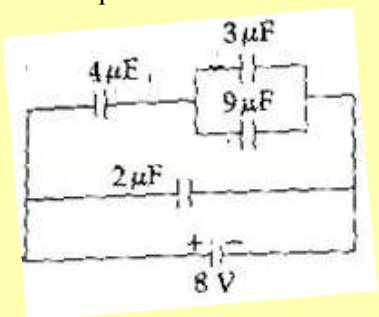
$$a + \frac{\ell\lambda}{a} \geq \sqrt{4\ell\lambda}$$

$$a = \frac{\ell\lambda}{a}$$

$$a = \sqrt{\ell\lambda}$$

$$b_{min} = \sqrt{4\ell\lambda}$$

74. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the 4μF and 12μF capacitors), at a point distant 30 m from it, would equal:



(1) 360 N/C (2) 420 N/C (3) 480 N/C (4) 240 N/C

Ans. (2)

$$4V_1 = 12V_2$$

$$\frac{V_1}{V_2} = \frac{3}{1}; \quad V_1 + V_2 = 8$$

$$V_1 = \frac{3}{4} \times 8 = 6 \text{ volts}$$

$V_2 = \frac{1}{4} \times 8 = 6$ volts

$Q_{4\mu F} = 24\mu C\left[C_1V_1\right] Q = Q_{4_r F} + Q_{9_r F} = 42\mu C$

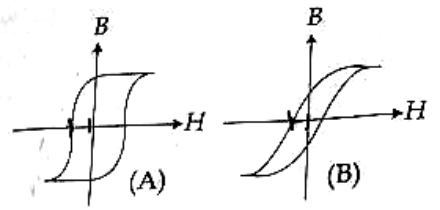
$Q_{9\mu F} = 18\mu C\left[C_2V_2\right] E = \frac{9\times10^9 \times 42\times10^{-6}}{9\times10^2}$

E = 420 N/C

75. Arrange the following electromagnetic radiations per quantum in the order of increasing energy:
A : Blue light B : Yellow light
C : X-ray D : Radiowave
(1) A, B, D, C (2) C, A, B, D (3) B, A, D, C (4) D, B, A, C

Ans. (4)

76. Hysteresis loops for two magnetic materials A and B are given below :



These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then it is proper to use:
(1) A for electromagnets and B for electric generators.
(2) A for transformers and B for electric generators.
(3) B for electromagnets and transformers.
(4) A for electric generators and transformers.

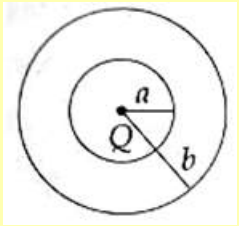
Ans. (3)
Transformers have lower hysteresis loss & electromagnets have lower retentivity

77. A pendulum clock loses 12 s a day if the temperature is 40°C and gains 4s a day if the temperature is 20°C. The temperature at which the clock will show correct time, and the co-efficient of linear expansion (α) of the metal of the pendulum shaft are respectively:

(1) $60°C;\alpha = 1.85\times10^{-4}/°C$ (2) $30°C;\alpha = 1.85\times10^{-3}/°C$
(3) $55°C;\alpha = 1.85\times10^{-2}/°C$ (4) $25°C;\alpha = 1.85\times10^{-5}/°C$

Ans. (4)

78. The region between two concentric spheres of radii 'a' and 'b', respectively (see figure), has volume charge density $\rho = \frac{A}{r}$, where A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant, is :



(1) $\frac{Q}{2\pi\left(b^2-a^2\right)}$ (2) $\frac{2Q}{\pi\left(a^2-b^2\right)}$ (3) $\frac{2Q}{\pi a^2}$ (4) $\frac{Q}{2\pi a^2}$

Ans. (4)

$$q_{enclosed} = Q + \int_a^x \frac{A}{r}.4\pi r^2 dr$$

$$= Q + 2\pi A\left(x^2 - a^2\right)$$

So at a distance x from centre.

$$E = \frac{1}{4\pi \in_0 . \frac{q_{enclosed}}{x^2}}$$

$$= \frac{1}{4\pi \in_0}\left[\frac{Q + 2\pi A\left(x^2 - a^2\right)}{x^2}\right]$$

$$= \frac{1}{4\pi \in_0}\left[2\pi A + \frac{Q - 2\pi A a^2}{x^2}\right]$$

If E is constant, then $Q - 2\pi A a^2 = 0$

$$\Rightarrow A = \frac{Q}{2\pi a^2}$$

79. In an experiment for determination of refractive index of glass of a prism by $i - \delta$, plot, it was found that a ray incident at angle $35°$, suffers a deviation of $40°$ and that it emerges at angle $79°$. In that case which of the following is closest to the maximum possible value of the refractive index?

(1) 1.6 (2) 1.7 (3) 1.8 (4) 1.5

Ans. (4)

$\delta = \ + \ - A$

$\Rightarrow \ \ = \ + \ - \delta = 74^0$

$\delta_{min} < \delta \Rightarrow \delta_{min} < 40^0$

$$\mu = \frac{\sin\left(\frac{A + \delta_{min}}{2}\right)}{\sin\frac{A}{2}} - \frac{\sin\left(\frac{\delta_m}{2} + 37^0\right)}{\sin 37^0} = \frac{5}{3}\sin\left(\frac{\delta_m}{2} + 37^0\right)$$

Now $\frac{\delta_m}{2} + 37^0 < \frac{40}{2} + 37^0 = 57^0$

$$\therefore \mu = \frac{5}{3}\sin\left(\frac{\delta_m}{2} + 37^0\right) < \frac{5}{3}\sin 57^0 \approx 1.4$$

$\therefore$ closest value is 1.5

80. A student measures the time period of 100 oscillations of a simple pendulum for times. The data set is 90s, 91s, 95 s and 92 s. If the minimum division in the measuring clock is 1s, then the reported mean time should be :

(1) $92 \pm 5.0\,s$ (2) $92 \pm 1.8\,s$ (3) $92 \pm 3\,s$ (4) $92 \pm 2\,s$

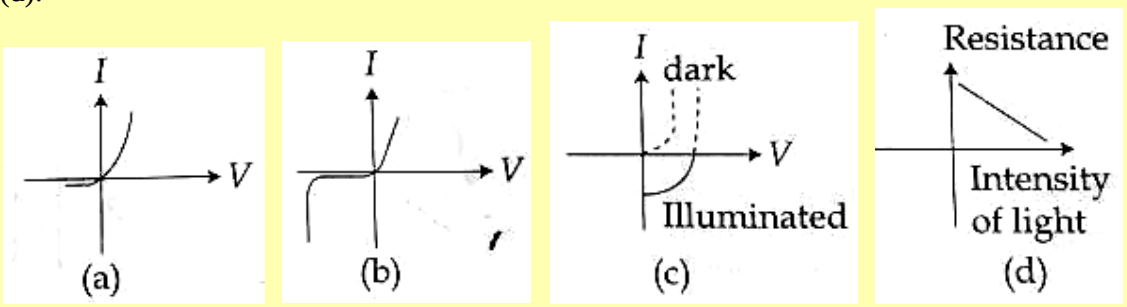
Ans. (4)

T (in s)	$\lvert T - T_{avg}\rvert$
90	2
91	1
95	3
92	0

$T_{avg} = 92$

$$\sum\left|T-T_{avg}\right|=1.5\approx 2s\left(\because \text{L.C.is}1s\right)$$
$$\therefore T=92\pm 2s$$

81. Identify the semiconductor devices whose characteristics are given below, in the order (a), (b), (c), (d):



(1) Zener diode, Simple diode, Light dependent resistance, Solar cell
(2) Solar cell, Light dependent resistanjce, Zener diode, Simple diode
(3) Zener diode, Solar cell, Simple diode, Light dependent resistance
(4) Simple diode, Zener diode, Solar cell, Light dependent resistance

Ans. (4)
Theoretical

82. Radiation of wavelength λ, is incident on a photocell. The fastest emitted electron has speed υ. If the wavelength is changed to $\frac{3\lambda}{4}$, the speed of the fastest emitted electron will be:

(1) $<\upsilon\left(\frac{4}{3}\right)^{\frac{1}{2}}$ (2) $=\upsilon\left(\frac{4}{3}\right)^{\frac{1}{2}}$ (3) $=\upsilon\left(\frac{3}{4}\right)^{\frac{1}{2}}$ (4) $>\upsilon\left(\frac{4}{3}\right)^{\frac{1}{2}}$

Ans. (4)

$$E-\phi=\frac{1}{2}mv_0^2$$
$$\frac{4E}{3}-\phi=\frac{1}{2}mv_1^2$$
$$\frac{4}{3}\left(\frac{1}{2}mv_0^2+\phi\right)\quad\frac{1}{2}mv_1^2+\phi$$
$$\frac{1}{2}m\left(\sqrt{\frac{4}{3}}v_0\right)^2+\frac{\phi}{3}=\frac{1}{2}mv_1^2+\phi$$
$$\frac{1}{2}m\left(\sqrt{\frac{4}{3}}V_0\right)^2+\left(\frac{4\phi}{3}\right)=\frac{1}{2}mv_1^2+\phi=E_1$$
$$\frac{\phi}{3}=\frac{1}{2}m\left(v_1^2-\sqrt{\frac{4}{3}}v_0^2\right)>0$$
$$v_1>\sqrt{\frac{4}{3}}v_0$$

83. A particle performes simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is :

(1) 3A (2) $A\sqrt{3}$ (3) $\frac{7A}{3}$ (4) $\frac{A}{3}\sqrt{41}$

Ans. (3)

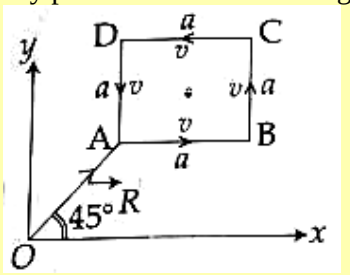
$V = \omega\sqrt{A^2 - x^2}$

So from question

$$\frac{3V}{V} = \frac{\sqrt{(A')^2 - \left(\frac{2A}{3}\right)^2}}{\sqrt{A^2 - \left(\frac{2A}{3}\right)^2}}$$

So $A' = \frac{7}{3}A$

84. A particle of mass m is moving along the side of a square of side 'a', with a uniform speed υ in the x–y plane as shown in the figure :



Which of the following statements is false for the angular momentum $\vec{L}$ about the origin?

(1) $\vec{L} = mv\left[\frac{R}{\sqrt{2}} - a\right]\hat{k}$ when the particle is moving from C to D.

(2) $\vec{L} = mv\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$ when the particle is moving from B to C.

(3) $\vec{L} = \frac{mv}{\sqrt{2}}R\hat{k}$ when the particle is moving from D to A.

(4) $\vec{L} = -\frac{mv}{\sqrt{2}}R\hat{k}$ when the particle is moving from A to B.

Ans. (1, 3)

$\vec{L} = \left[m\left(\vec{r} \times \vec{v}\right)\right]$

$\vec{L}_{CD} = mv\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$

$\vec{L}_{DA} = -\frac{mvR}{\sqrt{2}}\hat{k}$

$\vec{L}_{AB} = -\frac{mvR}{\sqrt{2}}\hat{k}$

$\vec{L}_{BC} = mv\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$

85. An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by PV^n =constant, then n is given by (Here C_P and C_V are molar specific heat at constant pressure and constant volume, respectively):

(1) $n=\frac{C-C_P}{C-C_V}$ (2) $n=\frac{C_P-C}{C-C_V}$ (3) $n=\frac{C-C_V}{C-C_P}$ (4) $n=\frac{C_P}{C_V}$

Ans. (1)

$$C=C_V+\frac{R}{(1-n)}$$

$$C(1-n)=C_V(1-n)+R$$

$$C=C_v+n(C-C_v)+R$$

$$\frac{(C-C_P)}{(C-C_v)}=n$$

86. A screw gauge with a pitch of 0.5 mm and a circular scale with 50 divisions is used to measure the thickness of a thin sheet of Aluminimum. Before starting the measurement, it is found that when the two jaws of the screw gauge are brought in contact, the 45th division coincides with the main scale line and that the zero of the main scale is barely visible. What is the thickness of the sheet if the main scale reading is 0.5 mm and the 25th division coincides with the main scale line?

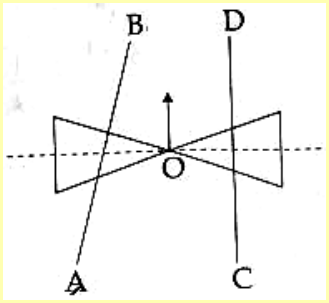
(1) 0.80 mm (2) 0.70 mm (3) 0.50 mm (4) 0.75 mm

Ans. (1)

Thickness of the sheet = $0.5+\left(30\times\frac{0.5}{50}\right)$

0.5 + 0.3 = 0.8 mm

87. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to :

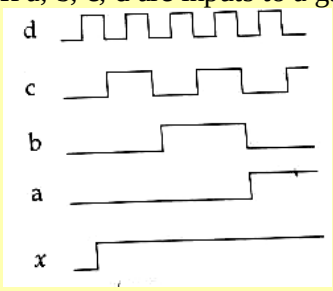


(1) turn right (2) go straight

(3) turn left and right alternately (4) turn left

Ans. (4)

88. If a, b, c, d are inputs to a gate and x is its output, then as per the following time graph, the gate is :



(1) AND (2) OR (3) NAND (4) NOT

Ans. (2)

a = 0
b = 0 $\Rightarrow x = 0$
c = 0
d = 0

a = 1
b = 0 $\Rightarrow x = 1$
c = 0
d = 0
and so on
So OR gate

89. For a common emitter configuration, if α β have their usual meanings, the incorrect relationship between α β is:

(1) $\alpha = \frac{\beta}{1-\beta}$ (2) $\alpha = \frac{\beta}{1+\beta}$ (3) $\alpha = \frac{\beta^2}{1+\beta^2}$ (4) $\frac{1}{\alpha} = \frac{1}{\beta} + 1$

Ans. (1 & 3)

$\frac{1}{\alpha} = \frac{1}{\beta} + 1$

$\therefore \alpha = \frac{\beta}{1+\beta}$

$\therefore$ (1) & (3) incorrect.

90. A satellite is revolving in a circular orbit at a height 'h' from the earth's surface (radius of earth R; h<<R). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to : (Neglect the effect of atmosphere.)

(1) $\sqrt{gR}$ (2) $\sqrt{gR/2}$ (3) $\sqrt{gR}\left(\sqrt{2}-1\right)$ (4) $\sqrt{2gR}$

Ans. (3)

$$\text{orbital speed} = \sqrt{\frac{GM}{R+h}}$$

$$= \sqrt{\frac{GM}{R}} \qquad \text{(as R >> h)}$$

$$= \sqrt{gR}$$

To escape to infinity from to close to earth surface.

Escape speed = $\sqrt{2gR}$

$\therefore \quad V_{req} = \sqrt{gR}\left(\sqrt{2}-1\right)$